American Railway Signaling Principles and Practices

CHAPTER XI

Alternating Current Track Circuits

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ALTERNATING CURRENT TRACK CIRCUITS

For many years direct current only was used for track circuits, but when it was found to be inadequate for electrified roads, due to the interference of the propulsion return current, the alternating current track relay was developed by Mr. J. B. Struble, about the year 1898. The alternating current track circuit consists of a portion of track sectionalized by means of insulated rail joints, as in direct current track circuits; its energy is derived either from a track transformer receiving energy from a line transformer connected to the transmission line or from a combined line and track transformer. The transformer connected to the track transmits its energy through the rails to an alternating current relay. In the track circuit are various auxiliary attachments which will be considered later.

The first relay designed for the alternating current track circuit was the single-element vane relay. It operates solely by the current received from the transformer through the rails of the track circuit, as shown in Fig. 1.

The development of the two-element alternating current relay followed several years after that of the single-element relay for use on long track circuits where the single-element relay is impractical due to the excessive power necessary to operate it.

The two-element track relay has a track element and a local element. The energy required in the track element is transmitted through the rails, and is comparatively small while the local element requires more energy, which is usually furnished from a local source. This relay requires the presence of currents in both elements at the same time to operate it as shown in Fig. 2. The absence of current in either element will de-energize the relay; thus a train entering the track circuit shunts the current from the track element de-energizing the relay.

Alternating current relays are practically immune to the propulsion current used by the railroad and to stray direct current. This latter feature along with their simplicity and their economy in maintenance has been the dominant factor in bringing the alternating track circuit into extensive use on steam roads. On electric roads the alternating track circuit is practically imperative, as the signal current and the return propulsion current flow through the same rails.

A low track voltage is necessary, as with direct current track circuits, to minimize the leakage of the track circuit current along the rails. The initial voltage depends on the type of relay, the length of track circuit and various other conditions.

A later development made in the interest of economy uses a transformer, rectifier and direct current relay in place of the alternating current relay.

Alternating Current Track Circuits on Steam Roads

Alternating current track circuits used on steam roads are generally end fed, as shown in Figs. 1 and 2.

On steam roads there are alternating current track circuits in service from a few feet up to approximately 10,000 feet in length. The maximum length of track circuit using a single-element relay is approximately 1000 feet.

Exhaustive studies indicate that where the minimum ballast resistance is not less than 4 ohms per 1000 feet of track that the maximum length for a 60-cycle track circuit using a two-element relay should not exceed 8000 feet. Where the minimum ballast resistance is as low as 2 ohms per 1000 feet the maximum length should not exceed 5500 feet. In 100-cycle territory these maximum lengths should be reduced to 6000 feet and 5000 feet respectively.

While track circuits equipped with two-element relays longer than those specified in the preceding paragraph may be operated, it is not advisable to do so for the following reasons:

1. It is impracticable to keep the voltage at the relay track element terminals within the limits to secure shunting value recommended by the Signal Section.
2. Added difficulty of wet weather operation.
3. Less broken-rail protection.

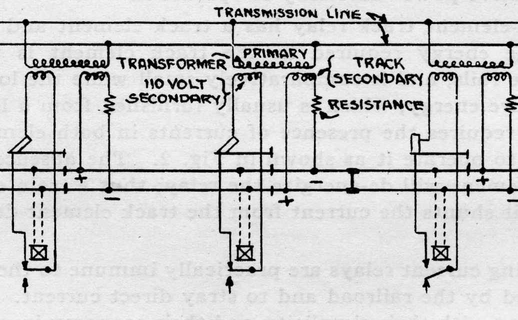

Fig. 1.
Single-Element Relay Track Circuits on Steam Roads.

The single-element track relay is a two-position relay, as shown in Fig. 1, while the two-element relay may be operated as a two-position relay as shown in Fig. 2, or as a three-position relay as shown in Fig. 3.

When the three-position relay is de-energized it assumes the central or neutral position, by means of a counterweight in the relay, as shown at A, in Fig. 3. The moving member makes contact to the right or left depending upon the relative flow of currents in the two elements of the relay. A reversal of the current flow in either element will reverse the movement of the magnetic flux. As the relative polarity of current in the local element is usually fixed, the flow of current in the track element is reversed by means of a pole changer operated by the signal mechanism, as shown in Fig. 3. It is this action of the pole changer that produces a change of phase angle of 180 degrees, as mentioned in Chapter X—Alternating Current Relays.

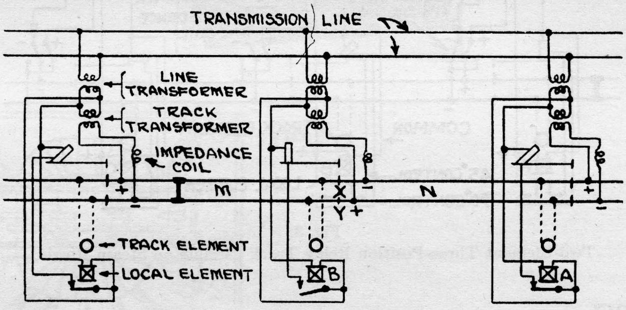

Fig. 2.
Two-Element Two-Position Relay Track Circuits on Steam Roads.

Insulated rail joint protection with two-element relays.

Some degree of protection against broken-down insulated joints is secured with two-element two-position alternating current relays by staggering or reversing the polarities of adjacent track circuits.

In Fig. 2, the polarities of the track circuits M and N are opposite at any given instant and if insulated joints X and Y were to break down, contacts of relay B should open due to a change in phase relation causing the signal to assume the Stop position. This same protection cannot be procured with the two-element three-position or the single-element relays, yet it is customary to stagger the polarities for a three-position relay in order to procure some protection by preventing the signal from displaying a more favorable indication than "approach." In Fig. 3, this is illustrated with the track circuits unoccupied and the polarities staggered. The polarities of the local element of alternate track relays as at B (Fig. 3) are reversed so that the same contacts may always be used for the Approach and Proceed indications respectively. Statements about changing and reversing polarity simply mean that at any given instant the polarities are reversed or changed from a positive to a negative polarity, since the alternating currents are periodically changing in direction.

While it is possible to feed two track circuits from one secondary of the transformer with the leads of the secondary in multiple, yet it is advisable to feed each track circuit from a separate secondary of the transformer. When one secondary is used to feed more than one track circuit a train occupying one track circuit may cause interference with the other track circuit, and also may lessen the broken-rail protection.

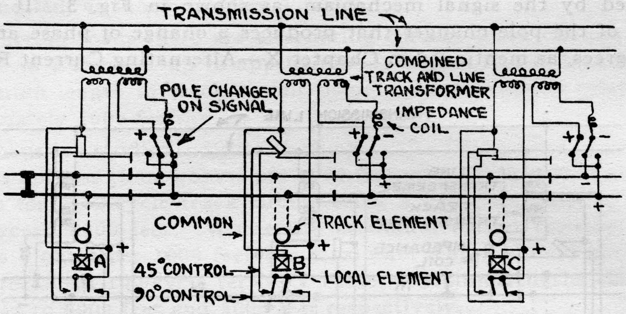

Fig. 3.
Two-Element Three-Position Relay Track Circuits on Steam Roads.

Reactors.

The track transformer having low internal resistance must have a reactor inserted between the transformer and the track to limit the flow of current when the track circuit is shunted, as the transformer might seriously heat or burn up, also power would be wasted. Adjustable track reactors are illustrated in Fig. 4.

The adjustable track reactor consists of a form wound coil well protected against moisture, assembled in a laminated iron core, divided into two parts, the space between them being adjustable. Further adjustment is made through different combinations of the various terminals. The adjustable track reactors are generally used on steam roads instead of resistors. The power factor varies from 0.1 to 0.3 and the voltage drop is practically wattless. With the single-element relay, it is used only as a matter of power economy as it has no bearing on its phase relation.

With the two-element relay, the adjustable reactor gives a very economical adjustment of the phase relation between the track and local elements. The relay operates most economically when the currents in the two elements are in ideal phase relation with each other. This ideal phase relation depends upon the type of relay, but usually it is approximately 90 degrees phase displacement. The phase displacement may be varied by adjusting the reactor. Track circuits should be adjusted so that the phase displacement does not vary more than 30 degrees from the ideal. This phase displacement at the relay varies with the ballast resistance and should, therefore, be adjusted to be as near the ideal as practicable under the least favorable ballast conditions when the track voltage is the lowest. When the ballast conditions are most favorable and the track voltage highest, the phase displacement will be the least favorable; thus the relay will be operating under the best possible conditions.

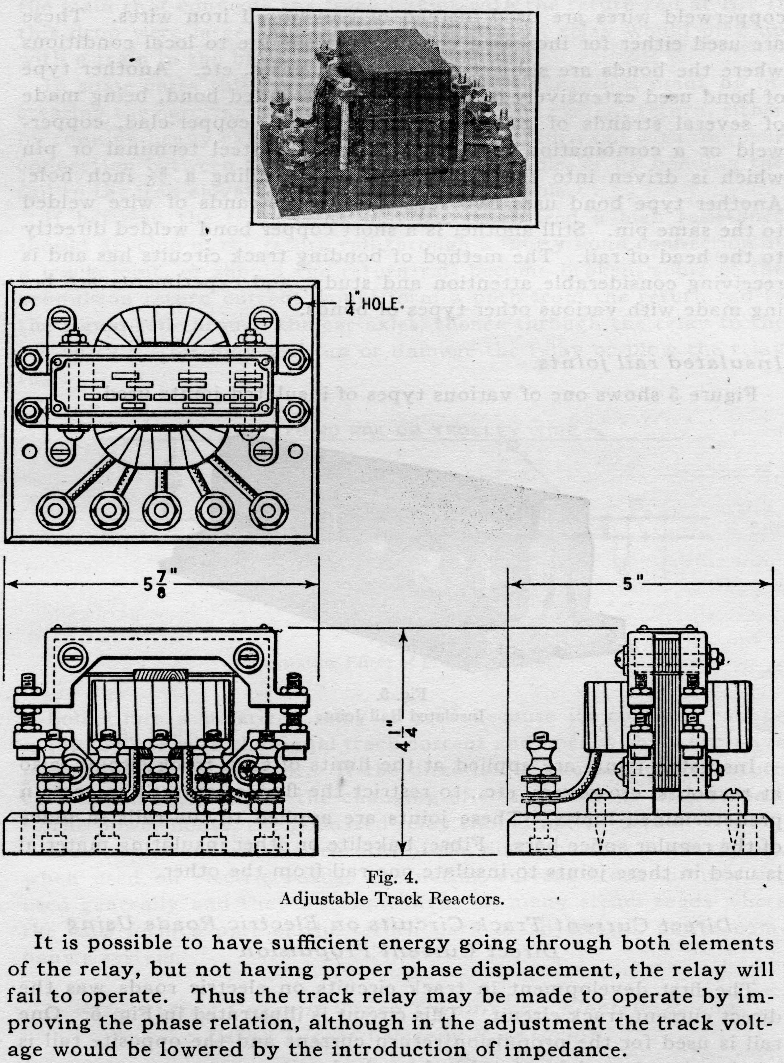

Fig. 4.
Adjustable Track Reactors.

It is possible to have sufficient energy going through both elements of the relay, but not having proper phase displacement, the relay will fail to operate. Thus the track relay may be made to operate by improving the phase relation, although in the adjustment the track voltage would be lowered by the introduction of impedance.

Sometimes when the track circuit is long and with poor ballast conditions, a non-adjustable reactor is used in the local element circuit. In some types of relays a small condenser is housed in the relay and used with the track element to aid in securing proper phase relations.

Bonding of steam road track circuits.

Bond wires are of different types, probably the most generally used being two or more No. 6 A.W.G. galvanized iron wires bonded to the web of each rail by channel pins. Frequently, copper, copper-clad or copperweld wires are used instead of galvanized iron wires. These are used either for increased conductivity or due to local conditions where the bonds are subject to salt brine, gases, etc. Another type of bond used extensively is known as the stranded bond, being made of several strands of galvanized iron, copper, copper-clad, copperweld or a combination of them welded to a steel terminal or pin which is driven into the web of rail after drilling a 3/8 inch hole. Another type bond uses two sets of several strands of wire welded to the same pin. Still another is a short copper bond welded directly to the head of rail. The method of bonding track circuits has and is receiving considerable attention and study, and experiments are being made with various other types of bonds.

Insulated rail joints.

Figure 5 shows one of various types of insulated joints used.

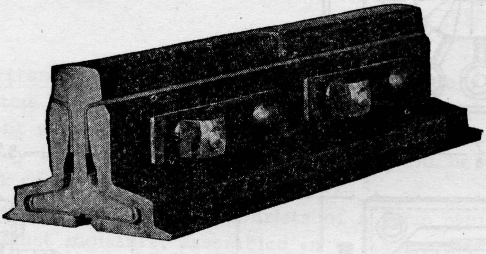

Fig. 5.
Insulated Rail Joint.

Insulated joints are applied at the limits of each track circuit, also at turnouts, crossovers, etc., to restrict the flow of current to certain predetermined limits. These joints are applied to the rails in place of the regular splice bars. Fibre, bakelite or other insulating material is used in these joints to insulate one rail from the other.

Direct Current Track Circuits on Electric Roads Using Direct Current Propulsion

The first development in track circuits on electric roads was the direct current track circuit. This circuit is illustrated in Fig. 6. One rail is used for the propulsion return current and the opposite rail is insulated and used for the signal track circuit.

With this arrangement there is a voltage drop all the way along the return rail proportional to the strength of the propulsion current and the resistance of the return rail, which resistance depends upon the size of the rail, the length of the track circuit and the track bonding. With a train at B moving in the direction of the arrow, a voltmeter (V) connected as shown on a direct current road will show a considerable direct current drop which will appear across terminals A and C of track relay X, due to the lower resistance in the axle of the train that connects the track circuit with the return rail at B. If this drop were large enough this direct current relay would pick up with a train in the block.

In some cases this drop is kept below the dangerous point by increasing the carrying capacity of the propulsion current return, either by running heavy cable in multiple with the return rail with frequent connection between the return rail and the cable, or by bonding the rail to the iron elevated structure where available.

If between the track relay and the track feed a high resistance should develop in the return rail through a faulty bond connection at a track joint, or should the return rail become broken, some of the propulsion return current would form a path from the return rail to the signal rail through the car axles, thence through the relay to the return rail; this may pick up or damage the relay or blow the relay fuse.

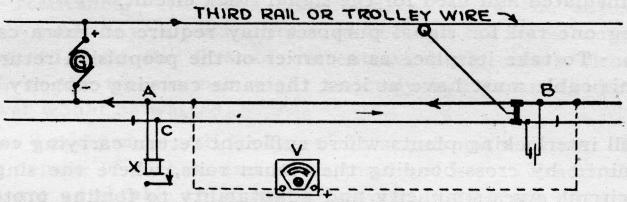

Fig. 6.
Illustrating Effect of Propulsion Drop.

Sometimes a polarized relay is used because its contacts can be made to close with the signal track current and open with an excessive propulsion return current. If the direction of the propulsion return current changes, due to the changing of train locations with respect to the power house, the polarized relay should not be used.

Due to the undesirable features of the direct current track circuit, when used on electric roads, alternating current track circuits are used generally and they also find favor on many steam roads where the rails form a part of the return path of an electric power company's system.

Alternating Current Track Circuits on Electric Roads Using Direct Current Propulsion

Generally, electric road alternating current track circuits and steam road alternating current track circuits are identical, except that on electric roads the return of the propulsion current must be given special consideration.

Single-rail track circuits.

The first development of the alternating current track circuit is shown in Fig. 7.

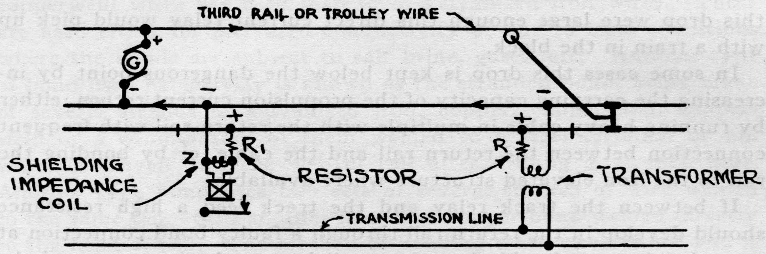

Fig. 7.
Single-Rail Alternating Current Track Circuit.

One rail is used for the propulsion return current and the opposite rail is insulated and used for the signal track circuit.

Using one rail for signal purposes may require an extra cable to be run. To take its place as a carrier of the propulsion return current this cable must have at least the same carrying capacity as the rail.

At all interlocking plants where sufficient return carrying capacity is obtained by cross-bonding the return rails, where the single-rail track circuit gives simplicity and adaptability to fouling protection, and where the installation of a two-rail track circuit would be costly and cumbersome, the single-rail track circuit has important advantages.

On a single-track road where one rail is used for the propulsion return current and the other for the signal track circuit, a measure of broken-rail protection is provided for both rails. On a multiple-track road where the rails used for the return of propulsion current are cross-bonded, it is possible, depending upon the system of cross-bonding, that a broken rail on the return propulsion current side of the track will not cause the relay to open and give the same signal protection as would be given on single-track road. A rail may be broken and still make sufficient contact with its other part, in which case the relay is not likely to open and give signal protection with either single or double-rail track circuits.

As in Fig. 6, a certain direct current voltage occurs across the relay terminals. Even when there is no train in the circuit, the signal rail is connected in multiple with the return rail by the transformer and relay track leads and carries a portion of the propulsion return current. The amount is inversely proportional to the resistance offered.

The direct propulsion current does not affect the safety of the alternating current track circuit as the relay is designed to operate only by alternating current. Since the ohmic resistance in the alternating current relay and the secondary of the transformer is quite low a considerable amount of direct propulsion current would flow through them were it not for the insertion of a resistance in the signal track circuit as shown at R and R_1 in Fig. 7.

Formerly, where the propulsion return current was less than 1500 amperes the tube resistor, Fig. 8, was used, but where the return current exceeded 1500 amperes, heavy cast-iron grid resistors were used at the transformer and the relay. The present practice is to use tubular resistors in all cases.

The grid resistor, Fig. 9, at the transformer not only reduces the return current but limits the flow of alternating current to the track when the track circuit is shunted.

Resistors.

As stated in the steam road section, track transformers have low internal resistance. All track transformers, except the adjustable filler type, must have a resistor inserted between the transformer and the track to limit the flow of current when the track circuit is shunted, as the transformer might seriously heat, causing damage, also power would be wasted.

Figures 8 and 9 show various types of track circuit resistors. These resistors consist of wire wound on an insulated core. Practically all of them are adjustable to adapt them to varying track circuit conditions. The size of the resistors used depends upon the amount of resistance required and the amount of current to be carried. They are practically non-inductive.

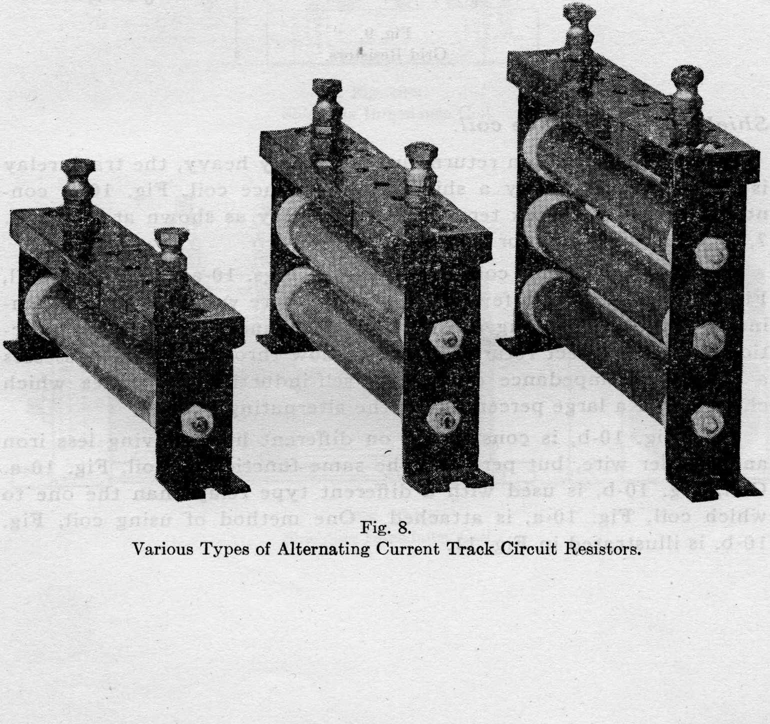

Fig. 8.
Various Types of Alternating Current Track Circuit Resistors.

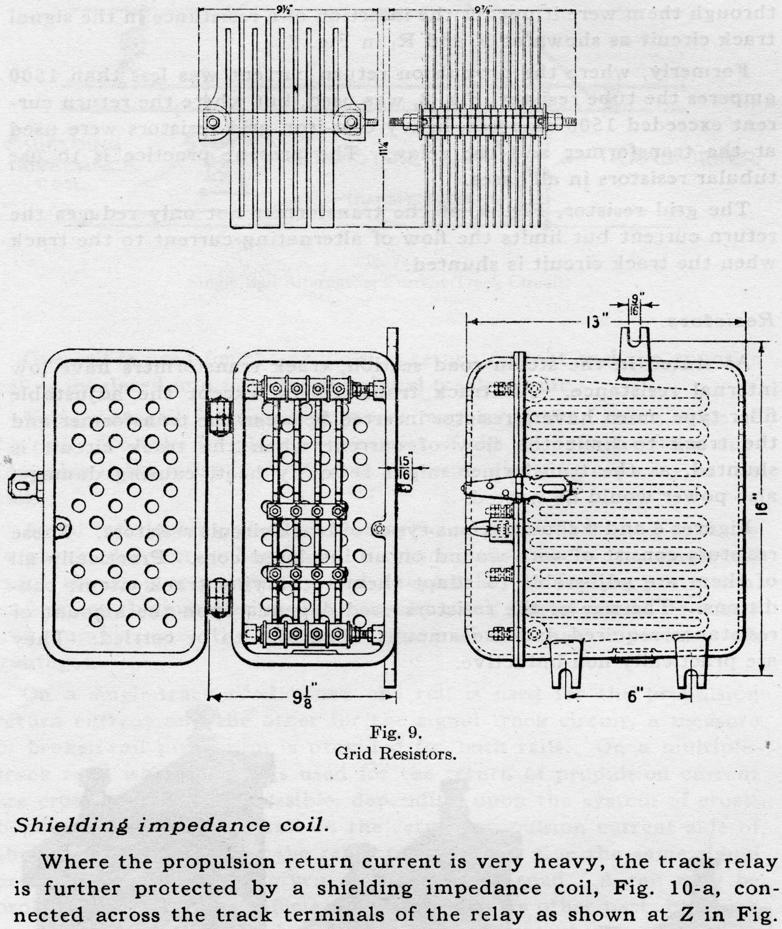

Fig. 9.
Grid Resistors.

Shielding impedance coil.

Where the propulsion return current is very heavy, the track relay is further protected by a shielding impedance coil, Fig. 10-a, connected across the track terminals of the relay as shown at *Z* in Fig. 7, to act as a by-pass for this current.

Shielding impedance coils are shown in Figs. 10-a and 10-b. Coil, Fig. 10-a, consists of a few turns of heavy wire wound around a laminated iron core, having a low ohmic resistance which allows practically all the direct return current to flow through it, but possesses a very high impedance due to its self-induction properties which chokes back a large percentage of the alternating current.

Coil, Fig. 10-b, is constructed on different lines, having less iron and smaller wire, but performs the same function as coil, Fig. 10-a. Coil, Fig. 10-b, is used with a different type relay than the one to which coil, Fig. 10-a, is attached. One method of using coil, Fig. 10-b, is illustrated in Fig. 11.

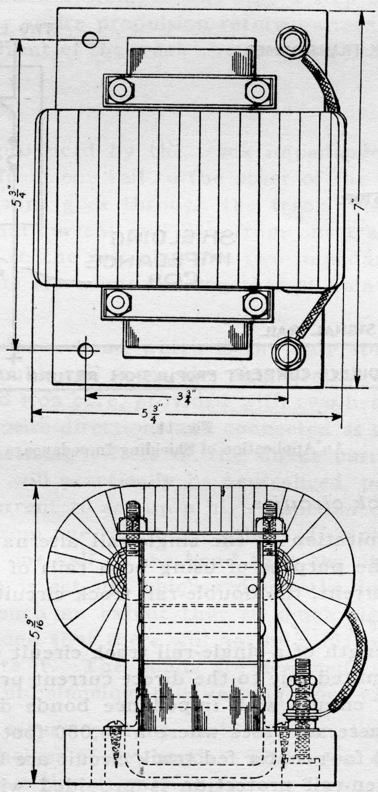

Fig. 10-a.
Shielding Impedance Coil.

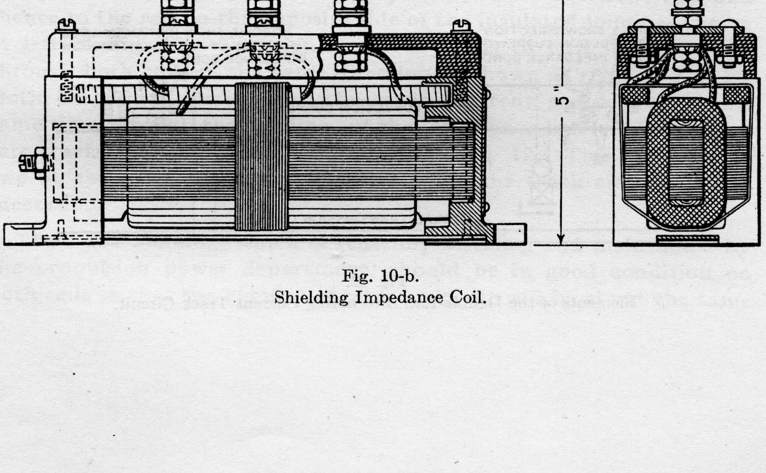

Fig. 10-b.
Shielding Impedance Coil.

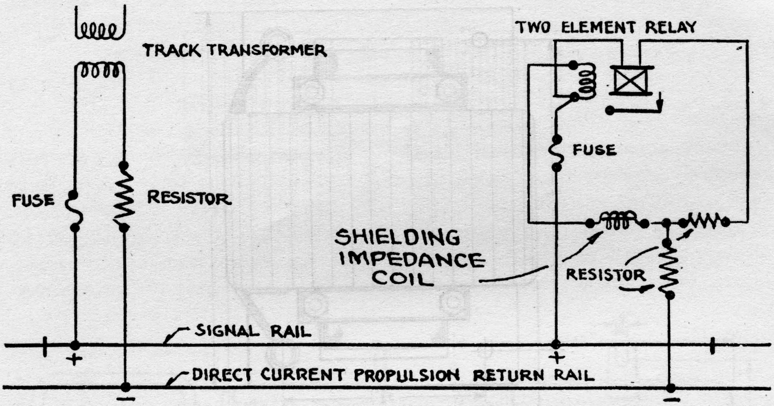

Fig. 11.
An Application of Shielding Impedance.

Double-rail track circuits.

Due to the limitation of the single-rail alternating current track circuit and for the purpose of using both rails of the track for propulsion return current, the double-rail track circuit shown in Fig. 12 was devised.

Whereas the length of a single-rail track circuit of a direct current electric road is limited due to the direct current propulsion drop, the double-rail track circuit with impedance bonds does not have this restriction and there are cases where a 20,000 foot end fed track circuit and a 25,000 foot center fed track circuit are being operated. A measure of broken-rail protection is provided with the double-rail track circuit. It is very stable and not easily affected by ballast leakage, due to the comparatively low impedance of the copper connection across the rails through the impedance bond.

The insulation of both rails is necessary as in the direct current track circuit, but in addition an impedance bond is used as shown at B in Fig. 12. After the impedance bond was developed for the relay

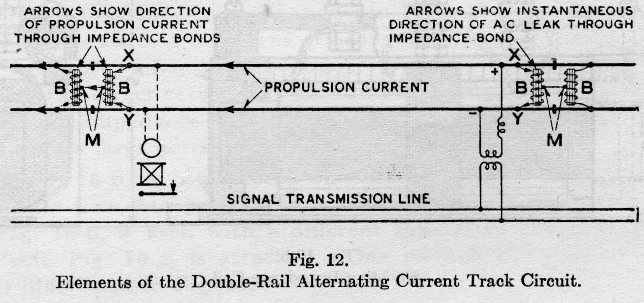

Fig. 12.
Elements of the Double-Rail Alternating Current Track Circuit.

in the single-rail track system, it was only a step to devise a larger device to carry the entire propulsion return current and choke back the alternating current of the track circuit by its inductance.

Impedance bonds.

The reactance produced by the track impedance bonds prevents excessive leakage from one rail to the other of the signal current so that sufficient current goes through the track relay to operate it. Practically no signal current will flow from one track circuit to the adjacent one, due to the fact that the two impedance bonds in adjacent track circuits are connected together at points of zero potential.

The track impedance bond which is the only special feature distinguishing the double-rail from the single-rail track circuit, consists of a laminated iron core, provided with two heavy copper windings wound in opposite directions and connected as shown in Fig. 12, so that the magnetizing effects of the direct current on the iron cores of the bonds will practically be neutralized, permitting the alternating signal current to set up a high impedance which prevents excessive leakage from one rail to the other. If more return current is flowing through one-half of the bond than the other half, an unbalanced condition is set up which reduces the impedance to the signal current to such an extent that so much signal current will pass through the bond that there will not be sufficient going through the relay to operate it. The bonds are designed to take care of a certain amount of unbalancing, this varying from 12 to 20 per cent.

The propulsion return current divides in multiple through the oppositely wound halves of the bonds. The signal current potential across the rails forces the alternating current through the two windings in series, but not in opposition, producing a reaction in both halves of the impedance bond, which permits sufficient current to go through the track relay so that it will pick up properly, the balance leaking through the impedance bond to the opposite rail.

The propulsion return current enters the bond at X and Y, Fig. 12, and passes to the other bond through the neutral connection M and thence to the rail on the opposite side of the insulated joints as shown at left of Fig. 12. The signal current tends to flow from X to Y through both coils in the same direction as shown at right of Fig. 12. Both the signal current and the return current are flowing at the same time through the windings of the impedance bonds. Four separate track impedance bonds are shown in Fig. 12. There are always two at each end of the track circuit when the track circuits are in succession.

The track bonding, which is generally installed and maintained by the propulsion power department, should be in good condition on both rails so that the amount of return current will be about the same in each rail, and the working condition of the impedance bonds will be good. Sometimes a track bond of one of the rails will become loose or broken, putting more resistance in one rail than in the other, causing an inequality of return current flow in the rails, which upsets the balancing effect of the impedance bonds. This excess current in one rail tends to magnetize the iron core, saturating it with direct current flux, lowering the permeability of the core and decreasing the reactance of the impedance bond, thus allowing a leakage of signal current through it. A heavy unbalancing return current will destroy the reactance of the bond and allow the signal current to pass freely from rail to rail as if a train were on the track section, de-energizing the relay and causing the signal to assume its most Restrictive indication. To minimize the effect of unbalancing, most impedance bonds are made with an air gap in the iron core to keep it from becoming saturated with direct current flux, but the reactance is lowered by the high reluctance of the air gaps. It is deemed the best practice to allow some leakage of signal current across the impedance bond to reduce the effect of the unbalancing return current.

Figure 13 shows the unbalancing curves of an impedance bond that will carry 2000 amperes per rail of propulsion return current. The air gap is $\frac{5}{64}$ inch.

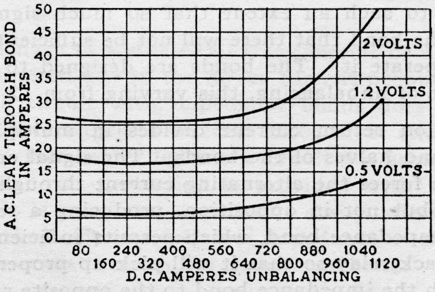

Fig. 13.

Unbalancing Curves for a 2000-Ampere Impedance Bond.

The vertical lines show the unbalancing direct current in amperes; the horizontal lines show the amount of signal current which will leak through the bond with voltage as indicated by curves. Note that the reactance is practically constant up to about 700 amperes unbalancing current, particularly on the lower voltage curves.

Figure 14 shows the inside construction of an impedance bond. The laminated iron core practically surrounds the heavy coils of bare copper wire. The bare wire is kept from touching the core and adjacent wires by wood or fibre strips used as spacers. The two central straps are connected together when the bond is completed and

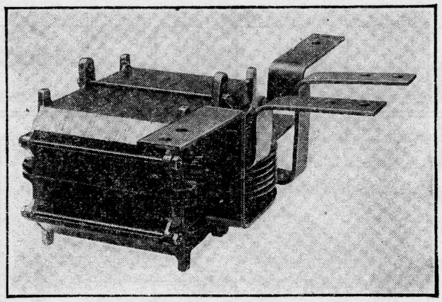

Fig. 14.
Core and Coils of an Impedance Bond.

form the neutral connector **M**. The two outside straps form the rail terminals **X** and **Y** shown in Fig. 12. The air gap between the two iron cores is faintly shown as a horizontal white line on the face of the core.

The impedance bond shown in Fig. 14, when in service, is inclosed in an iron case generally filled with petrolatum or solid vaseline compound to protect the bond from dampness.

Figure 15 shows two bonds installed in a double-rail track circuit. This type has 1500-ampere capacity per rail and is suitable for heavy traction lines. Impedance bonds are designed for various capacities and are installed either between the rails or at the side of the track as illustrated in Fig. 16.

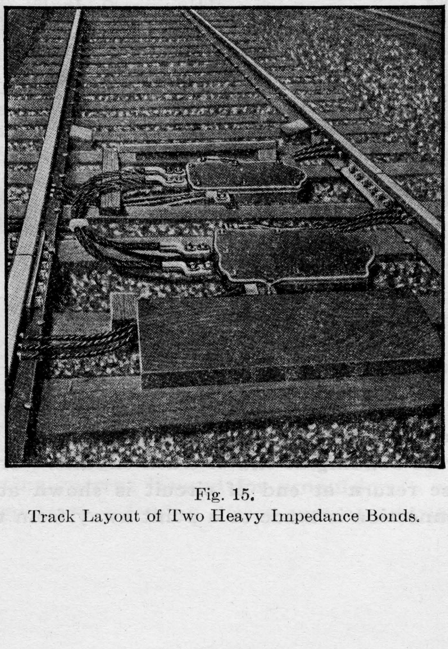

Fig. 15.
Track Layout of Two Heavy Impedance Bonds.

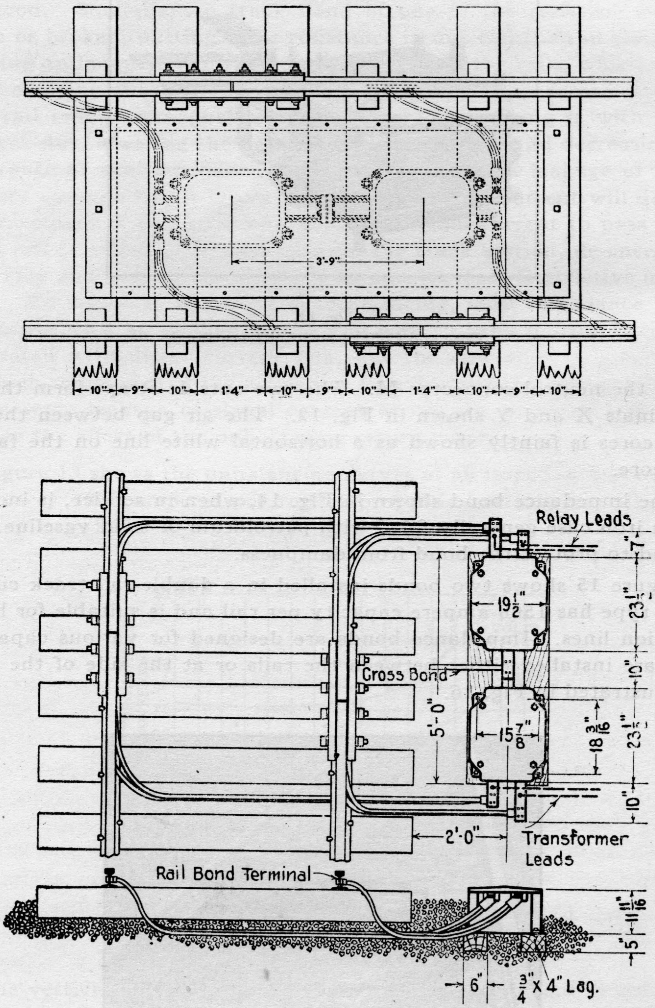

Fig. 16.
Methods of Installing Impedance Bonds.

Cross-bonding.

Methods of cross-bonding, insulating and bonding of sidings and making connection to power house return are shown in Fig. 17. B and C show cross-bonding at end of track circuits. Cross connection to power house return at end of circuit is shown at A. E shows where cross connection is made at a point away from the ends of the

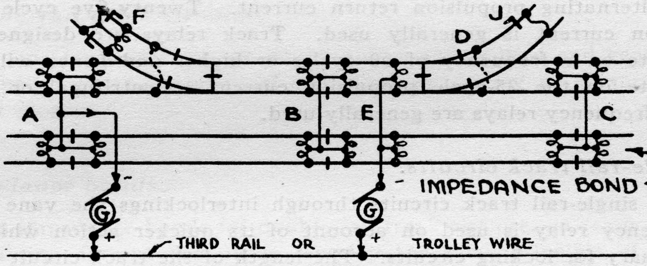

Fig. 17.
Cross-Bonding Connections.

track circuit. In this case supplementary single impedance bonds are inserted. This method is undesirable, not only due to the additional cost, but because of the extra leak in the track circuit.

Relays.

The galvanometer, induction motor or two-element vane type relays are generally used as on steam roads or on single-rail track circuits of electric traction roads, although they are wound for a lower voltage and higher current in order to keep down the leakage of signal current across the track through the impedance bonds.

Transformers.

Either adjustable filler type or regular track transformers are used but with greater capacity than those used on steam roads, due to the extra amount of signal current that leaks through the impedance bonds.

Reactors and resistors.

Reactors instead of resistors may be used between the transformer and the track when required. Reactors are more economical than resistors and are generally required between the transformer and the track where galvanometer track relays are used in order to bring the track current in phase with the local current. Resistors give best results between the transformer and track where induction motor type track relays are used.

Alternating Current Track Circuits on Electric Roads Using Alternating Current Propulsion

Relays.

When alternating current is used for propulsion, a relay must be used that is immune to a foreign direct current and inoperative by the alternating propulsion return current. Twenty-five cycle propulsion current is generally used. Track relays are designed to operate on a frequency of 60 cycles or higher and hence will not operate on the 25-cycle propulsion current. Centrifugal or vane type frequency relays are generally used.

Single-rail track circuits.

On single-rail track circuits through interlockings the vane type frequency relay is used on account of its quicker action which is necessary for locking circuits. The length of the track circuit must be restricted on account of the drop in the propulsion current causing injurious heating current to pass through the track relay or transformer.

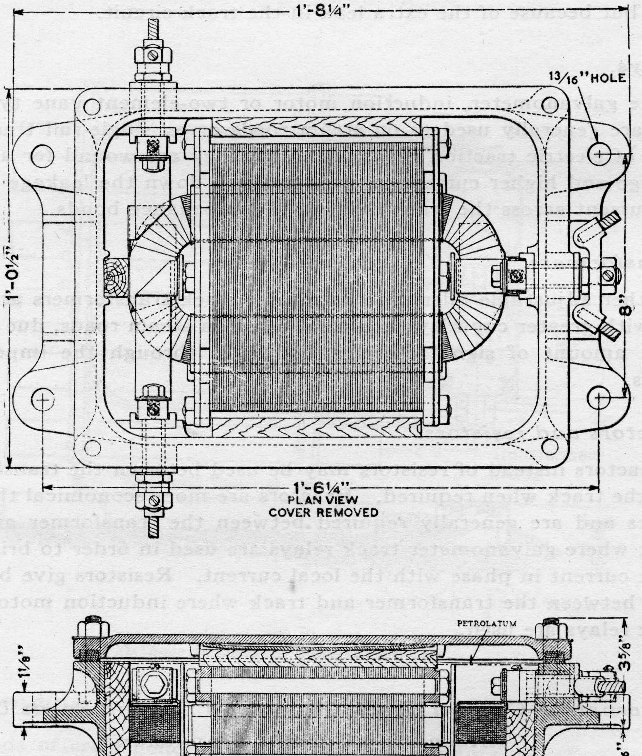

Fig. 18.
Single Impedance Bond for Alternating Current Propulsion.

Double-rail track circuits.

On long track circuits the double-rail circuit is used and generally equipped with the centrifugal type frequency relay as a matter of power economy.

Impedance bonds.

The impedance bonds for double-rail track circuits, on roads using alternating current propulsion, employ the same principle of magnetic balancing as characterizes the bonds for direct current roads, for, although the propulsion current is of an alternating character, it is divided between the two opposing windings of the bonds, so that the alternating magneto-motive forces are equal and opposite, and hence neutralize each other. The iron core of the bond remains, therefore, unmagnetized, so that it offers a high permeability to the magneto-motive force generated by the alternating signal current flowing through the two coils in series, as with the bonds for direct current propulsion.

Figure 18 shows a single impedance bond used on alternating current roads.

These bonds are smaller than those used on direct current propulsion roads and sometimes two bonds are used in one housing as shown in Fig. 19. Whereas on direct current propulsion roads the return current may run as high as 2500 amperes per rail, that on alternating current propulsion roads will generally be between the limits of 75 and 700 amperes per rail, due to the higher alternating current propulsion voltages ordinarily used. This allows the impedance bonds to be made considerably smaller.

Unbalancing.

Unbalancing troubles are rare on roads using alternating current propulsion, not only because the propulsion currents themselves are small in volume, but especially because, if more current flows in one-half of the bond than the other, the half winding carrying the heavier current induces a voltage in the weaker half, tending to pull a larger current through that weaker half. Thus an automatic action exists which tends to keep the bond well balanced. For this reason, the bonds are not liable to be unbalanced, and no air gap is required in the magnetic circuit to prevent saturation of the core which would otherwise occur.

Fuses.

On electric roads fuses are usually installed between relays and rails to protect relays from excessive current.

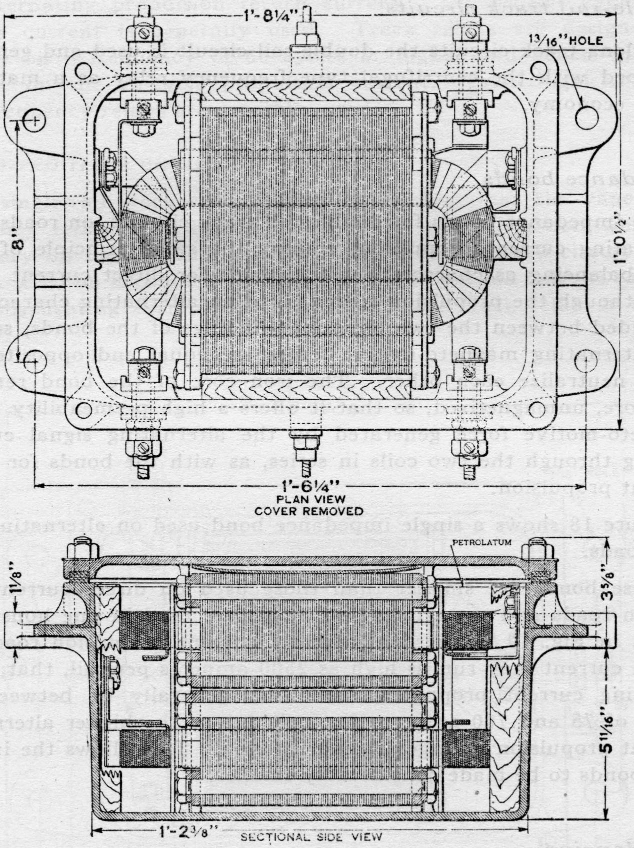

Fig. 19.
Double Impedance Bonds for Alternating Current Roads.

Track Circuit Calculations for Alternating Current Signal System

General.

The proper calculation of the track circuit is of prime importance in the design of an alternating current signal system as it enables the engineer to select that type of track circuit apparatus which will operate most economically under the particular set of conditions in question. Furthermore, aside from the matter of economy, maximum safety of the track circuit can only be guaranteed by proper track circuit adjustments as dictated by the calculations. The formulae and diagrams here presented should enable the reader to make a full analysis of any type of circuit operating under any conditions he may encounter.

Resistance, reactance and impedance of rails.

The track circuit is in reality a small single-phase transmission system whose two line wires are represented by the rails, and whose load is represented by the relay at the end of the track circuit. Like the line wires of the transmission, the rails possess impedance (Z) composed of resistance (R) and reactance (X); the effective resistance of a steel rail is, however, from three to five times the actual resistance to direct current, due to the fact that the flow of alternating current in the magnetic material of which the rail is composed, sets up a magnetic field producing a counter *e.m.f.* in the body of the rail itself, forcing the current to the outer surface or skin of the rail, rendering thereby but a fraction of the cross-sectional area available for conducting current. This is known as the "skin effect," and is present in a greater or less degree in all conductors carrying alternating currents.

A further increase in the apparent resistance of the rails is introduced by their self-inductance, this depending on the spacing of the rails and their size, just as in the case of the two wires of a transmission. Since the rails are magnetic, however, their respective fields will be considerably more localized around each conductor than would be the case if non-magnetic conductors were in question, and hence the reactance of the rail circuit will be much greater than would be indicated by the usual formulae and tables for non-magnetic conductors. The "skin effect" depends upon the permeability of the rails, which latter factor is a variable depending on the current density. Due to the presence of this variable, the magnitude of the skin effect is not susceptible to mathematical calculation. The permeability factor also obviously influences the magnitude of the self-inductance and in turn the reactance of the circuit.

Actual measurements have, therefore, had to be resorted to, and Table I so determined gives the total impedance per 1000 feet of track (both rails including bond wires) under various conditions of bonding in common practice, and for values of current commonly used for relay energization; this table has been in use for several years and has been found to give results sufficiently accurate for all practical purposes. Where 39-foot rails are in service the figures shown for 33-foot rails may be used for calculating purposes. While the values shown apply especially to steam road conditions, they may be used for electric road track circuit calculations, since the presence of propulsion current in the rails will only tend to decrease the permeability of the rails and in turn their effective resistance and impedance; hence the voltage at the relay may increase slightly with heavy propulsion currents flowing in the rails and any error introduced will be on the safe side. Table II shows separately the resistance of various kinds of bond wires as used in steam road work; on electric roads the rail is bonded to capacity, or nearly so, for propulsion current.

TABLE I

Impedance of Bonded Rails to Signal Currents in Ohms per 1000 Feet of Track

Weight of rail, lb. per yd.	Bonding	30 ft. rails						33 ft. rails					
		25 cycle		60 cycle		100 cycle		25 cycle		60 cycle		100 cycle	
		Z	P.F.	Z	P.F.	Z	P.F.	Z	P.F.	Z	P.F.	Z	P.F.
130	To capacity	0.10	0.35	0.19	0.27	0.30	0.23	0.10	0.35	0.19	0.27	0.30	0.23
	2 No. 6 copper	0.12	0.69	0.21	0.49	0.31	0.38	0.12	0.68	0.21	0.47	0.31	0.37
	1 No. 8 iron) 1 No. 6 copper)	0.15	0.81	0.23	0.60	0.33	0.47	0.15	0.79	0.23	0.58	0.32	0.46
	2 No. 6 copper-clad 40%.	0.17	0.85	0.25	0.66	0.34	0.53	0.16	0.84	0.24	0.64	0.34	0.51
	2 No. 6 copper-clad 30%.	0.20	0.90	0.27	0.74	0.36	0.60	0.19	0.89	0.27	0.71	0.35	0.58
	2 No. 8 iron	0.36	0.97	0.41	0.89	0.48	0.80	0.34	0.97	0.39	0.88	0.46	0.78
120	To capacity	0.10	0.36	0.20	0.28	0.30	0.23	0.10	0.36	0.20	0.28	0.30	0.23
	2 No. 6 copper	0.13	0.69	0.22	0.49	0.32	0.38	0.12	0.68	0.21	0.48	0.32	0.37
	1 No. 8 iron) 1 No. 6 copper)	0.15	0.81	0.24	0.61	0.34	0.48	0.15	0.79	0.23	0.59	0.33	0.45
	2 No. 6 copper-clad 40%.	0.17	0.85	0.25	0.66	0.35	0.53	0.17	0.83	0.25	0.64	0.34	0.51
	2 No. 6 copper-clad 30%.	0.21	0.90	0.28	0.74	0.37	0.59	0.20	0.89	0.27	0.71	0.36	0.57
	2 No. 8 iron	0.36	0.97	0.41	0.89	0.49	0.79	0.34	0.96	0.39	0.88	0.47	0.77
110	To capacity	0.10	0.37	0.20	0.29	0.31	0.24	0.10	0.37	0.20	0.29	0.31	0.24
	2 No. 6 copper	0.13	0.69	0.22	0.49	0.33	0.39	0.13	0.67	0.22	0.48	0.32	0.38
	1 No. 8 iron) 1 No. 6 copper)	0.16	0.80	0.24	0.60	0.34	0.48	0.15	0.78	0.24	0.58	0.34	0.47
	2 No. 6 copper-clad 40%.	0.18	0.84	0.26	0.66	0.36	0.53	0.17	0.83	0.25	0.64	0.35	0.51
	2 No. 6 copper-clad 30%.	0.21	0.89	0.29	0.73	0.38	0.60	0.20	0.88	0.28	0.71	0.37	0.58
	2 No. 8 iron	0.37	0.96	0.42	0.88	0.49	0.79	0.34	0.96	0.40	0.87	0.47	0.77

	To capacity	0.11	0.38	0.21	0.30	0.32	0.25	0.11	0.38	0.21	0.30	0.32	0.25
	2 No. 6 copper	0.14	0.69	0.23	0.49	0.34	0.40	0.13	0.67	0.23	0.48	0.34	0.38
	1 No. 8 iron) 1 No. 6 copper).........	0.16	0.79	0.25	0.60	0.36	0.48	0.16	0.78	0.25	0.58	0.35	0.46
100	2 No. 6 copper-clad 40%.	0.18	0.84	0.27	0.65	0.37	0.53	0.17	0.83	0.26	0.63	0.36	0.51
	2 No. 6 copper-clad 30%.	0.22	0.89	0.29	0.73	0.39	0.60	0.20	0.87	0.29	0.70	0.38	0.57
	2 No. 8 iron	0.37	0.96	0.43	0.88	0.51	0.78	0.35	0.96	0.41	0.87	0.49	0.76
	To capacity	0.11	0.39	0.22	0.30	0.33	0.26	0.11	0.39	0.22	0.30	0.33	0.26
	2 No. 6 copper	0.14	0.68	0.24	0.50	0.35	0.39	0.14	0.67	0.24	0.48	0.35	0.38
	1 No. 8 iron) 1 No. 6 copper).........	0.17	0.79	0.26	0.60	0.36	0.47	0.16	0.78	0.26	0.58	0.36	0.46
90	2 No. 6 copper-clad 40%.	0.19	0.83	0.27	0.66	0.38	0.52	0.18	0.82	0.27	0.64	0.37	0.50
	2 No. 6 copper-clad 30%.	0.22	0.88	0.30	0.73	0.40	0.59	0.21	0.87	0.29	0.70	0.39	0.57
	2 No. 8 iron	0.37	0.96	0.43	0.88	0.51	0.78	0.35	0.96	0.41	0.87	0.49	0.76
	To capacity	0.11	0.39	0.22	0.31	0.34	0.26	0.11	0.39	0.22	0.31	0.34	0.26
	2 No. 6 copper	0.14	0.68	0.24	0.49	0.35	0.39	0.14	0.67	0.24	0.47	0.35	0.38
	1 No. 8 iron) 1 No. 6 copper).........	0.17	0.79	0.26	0.59	0.37	0.47	0.16	0.78	0.26	0.57	0.37	0.46
85	2 No. 6 copper-clad 40%.	0.19	0.83	0.28	0.64	0.38	0.52	0.18	0.82	0.27	0.62	0.38	0.50
	2 No. 6 copper-clad 30%.	0.22	0.88	0.30	0.72	0.40	0.59	0.21	0.87	0.30	0.70	0.40	0.56
	2 No. 8 iron	0.38	0.96	0.44	0.87	0.52	0.78	0.35	0.95	0.42	0.86	0.50	0.76
	To capacity	0.12	0.39	0.23	0.31	0.34	0.26	0.12	0.39	0.23	0.31	0.34	0.26
	2 No. 6 copper	0.15	0.68	0.25	0.49	0.36	0.40	0.14	0.66	0.25	0.48	0.36	0.39
	1 No. 8 iron) 1 No. 6 copper).........	0.17	0.78	0.27	0.60	0.38	0.48	0.17	0.77	0.26	0.58	0.38	0.46
80	2 No. 6 copper-clad 40%.	0.19	0.83	0.28	0.64	0.39	0.53	0.18	0.81	0.28	0.63	0.39	0.51
	2 No. 6 copper-clad 30%.	0.22	0.88	0.31	0.71	0.41	0.59	0.21	0.86	0.30	0.70	0.40	0.57
	2 No. 8 iron	0.38	0.96	0.44	0.87	0.53	0.77	0.35	0.95	0.42	0.86	0.51	0.75
	To capacity	0.12	0.40	0.24	0.32	0.36	0.27	0.12	0.40	0.24	0.32	0.36	0.27
	2 No. 6 copper	0.15	0.67	0.26	0.49	0.38	0.40	0.15	0.66	0.26	0.48	0.37	0.39
	1 No. 8 iron) 1 No. 6 copper).........	0.18	0.78	0.28	0.59	0.39	0.48	0.17	0.76	0.28	0.57	0.39	0.46
70	2 No. 6 copper-clad 40%.	0.20	0.82	0.30	0.64	0.41	0.52	0.19	0.81	0.29	0.62	0.40	0.50
	2 No. 6 copper-clad 30%.	0.23	0.87	0.32	0.71	0.43	0.58	0.22	0.86	0.31	0.68	0.42	0.56
	2 No. 8 iron	0.38	0.96	0.45	0.86	0.54	0.77	0.36	0.95	0.43	0.85	0.52	0.75

TABLE II

Resistance of Bonds to Signal Currents

Ohms per 1000 feet of track

Bonds per joint	27.5 ft. rails	30 ft. rails	33 ft. rails	
2 No. 6 B&S copper	0.057	0.052	0.048	Bonds 48 inches long; no allowance is made for conductance of fish plates.
1 No. 6 B&S copper & 1 No. 8 BWG iron	0.098	0.089	0.082	
2 No. 6 copper-clad 40%	0.124	0.112	0.103	
2 No. 6 copper-clad 30%	0.166	0.150	0.138	
2 No. 8 BWG iron	0.348	0.315	0.291	

Ballast, leakage resistance and conductance.

The resistance of the leakage path between rails in ohms per 1000 feet of track varies with the nature of the ballast, the condition of the ties, and the weather conditions. In connection with the calculations involving rail impedance as given in Tables I and II, the following values for resistance of ballast and ties may be used; they are given for ballast cleared away from the rails:

	Ohms per 1000 ft. of track
Wet gravel	2
Dry gravel	3
Wet broken stone	4
Dry broken stone	10

In making track circuit calculations, a leakage resistance of 4 ohms per 1000 feet is very commonly used as representing the worst condition of well-drained broken stone or rock ballast; 2 ohms per 1000 feet is a low wet weather value for track with gravel ballast. Poorly drained cinder ballast with old water-soaked ties will run generally as low as 1 ohm per 1000 feet. In making the calculations the wet weather ballast leakage figure should be used as if the track transformer were designed and track circuit adjustments were made on the dry weather basis, the track relay might fail to pick up in wet weather. It is, however, advisable to make a check calculation on the dry weather basis in order to determine the variation in voltage on the track relay from the wet to the dry condition, as in the case of extremely long track circuits with poor ballast, the relay voltage in dry weather may be so high that special means may have to be employed to prevent the relay from being excessively energized. In track circuit calculations it is generally more convenient to represent the ballast leakage factor in terms of conductance rather than resistance; conductance (expressed in mhos) is the inverse of resistance (expressed in ohms), and thus a ballast leakage resistance of 4 ohms per 1000 feet corresponds to a ballast conductance of ¼ mho per 1000 feet.

Track circuit formulae and their derivation.

Given the voltage *e* and the current *i* required at the track relay terminals, the length of the block, the rail impedance with its power factor, and the ballast leakage resistance, the problem which confronts us is the determination of the power to be fed into the track at the transformer end.

To begin with, due to the impedance drop in the rails caused by the relay current, the difference of potential between the rails increases from *e* at the relay end of the track circuit to some higher value E at the transformer end; thus, the ballast leakage current increases as we proceed from the relay to the transformer. The ballast leakage current itself produces a drop in the rails which again increases the voltage required at the transformer. The fact that the ballast conductance is distributed uniformly throughout the length of the track circuit rather complicates matters in that the current in the rails and the voltage across them from point to point changes with the varying magnitude of the ballast leakage current. In order to simplify matters, it is sometimes assumed that the ballast leak is concentrated at the center of the track circuit, but this is not strictly accurate; evidently the concentrated ballast leak is located nearer the transformer end of the track circuit than the relay end, for it is near the transformer end that the voltage is highest and the ballast leakage greatest. The correct determination of the ballast leak is therefore somewhat of an involved process. It can, however, be determined, and in fact this method is quite extensively used in England; the reader who is interested in this phase of the matter is referred to a very interesting and complete discussion given in the January 1915 issue of the *Railway Engineer of London.*

It is evidently more accurate to consider the ballast conductance as uniformly distributed, and by means of the following formulae, originated by Mr. L. V. Lewis and first presented in the July 1911 number of the *Signal Engineer,* the voltage E and the current I at the transformer end of the track circuit, as well as their phase relationship, can easily be calculated. These general equations are:

$$E = e \cosh \sqrt{ZG} + i \sqrt{\frac{Z}{G}} \sinh \sqrt{ZG} \qquad (1)$$

$$I = i \cosh \sqrt{ZG} + e \sqrt{\frac{G}{Z}} \sinh \sqrt{ZG} \qquad (2)$$

where *e* and *i* are the relay voltage and current respectively; *Z* is the total impedance of the rails of the track secured by multiplying the values in Table I by the length of the track circuit in thousands of feet, and G is the total ballast leakage conductance secured by multiplying the reciprocal of the ballast leakage resistance in ohms per thousand feet by the length of the track circuit in thousands of feet. The terms *cosh* and *sinh* (pronounced "cosh" and "shin") are the hyperbolic cosine and sine respectively of an imaginary or complex angle represented in this case by the quantity $\sqrt{ZG}$. These formulae may be reduced to workable form by expanding the functions into their corresponding infinite series beginning

$$\cosh x = 1 + \frac{x^2}{\lfloor\!\underline{2}} + \frac{x^4}{\lfloor\!\underline{4}} + \frac{x^6}{\lfloor\!\underline{6}} + \dots \quad (3)$$

$$\sinh x = x + \frac{x^3}{\lfloor\!\underline{3}} + \frac{x^5}{\lfloor\!\underline{5}} + \frac{x^7}{\lfloor\!\underline{7}} + \dots \quad (4)$$

where x represents the hyperbolic angle $\sqrt{ZG}$ and the sign $\lfloor\!\underline{\ \ }$ represents arithmetical multiplication; for example, $\lfloor\!\underline{3}$ is called "factorial three" and is equal 1 x 2 x 3 = 6; likewise, $\lfloor\!\underline{4} = 1$ x 2 x 3 x 4 = 24. Hence

$$\cosh \sqrt{ZG} = 1 + \frac{ZG}{2} + \frac{(ZG)^2}{24} + \frac{(ZG)^3}{720} + \dots \quad (5)$$

$$\sinh \sqrt{ZG} = \sqrt{ZG} + \frac{\sqrt{(ZG)^3}}{6} + \frac{\sqrt{(ZG)^5}}{120} + \dots \quad (6)$$

Substituting the above values in equations (1) and (2)

$$E = \left(e + \frac{eZG}{2} + \frac{e(ZG)^2}{24} + \frac{e(ZG)^3}{720} + \right) + \left(i\sqrt{\frac{Z}{G}}\sqrt{ZG} + \frac{i\sqrt{\frac{Z}{G}}\sqrt{(ZG)^3}}{6} + \frac{i\sqrt{\frac{Z}{G}}\sqrt{(ZG)^5}}{120} + \right) \quad (7)$$

$$I = \left(i + \frac{iZG}{2} + \frac{i(ZG)^2}{24} + \frac{i(ZG)^3}{720} + \right) + \left(e\sqrt{\frac{G}{Z}}\sqrt{ZG} + \frac{e\sqrt{\frac{G}{Z}}\sqrt{(ZG)^3}}{6} + \frac{e\sqrt{\frac{G}{Z}}\sqrt{(ZG)^5}}{120} + \right) \quad (8)$$

Reducing and rearranging the terms of equation (7) and (8)

$$E = e + Zi + \frac{Z}{2}Ge + \frac{Z}{3}\,\frac{G}{2}Zi + \frac{Z}{4}\,\frac{G}{3}\,\frac{Z}{2}Ge + \dots \quad (9)$$

$$I = i + Ge + \frac{G}{2}Zi + \frac{G}{3}\,\frac{Z}{2}Ge + \frac{G}{4}\,\frac{Z}{3}\,\frac{G}{2}Zi + \dots \quad (10)$$

The above equations may be carried out to any number of terms by carrying out the above process, noting that the first element of each term of equation (9) is *Z*, and of equation (10) G, each divided by 1, 2, 3, 4, etc., according to the number of the term in the infinite series, the remaining elements of the term under consideration being identical with the next preceding term in the other equation. Sufficient accuracy for all practical purposes will in most cases be secured by calculating only the first five terms of each series as above shown, the remaining terms being generally small enough in value to be disregarded.

Equations (9) and (10) may also be developed direct from Ohm's law, stating that $E = I\,Z$ and $I = E\,G$, and consideration of the matter on this basis will enable the reader to grasp fully their physical meaning. To begin with, the first two terms e and i of equations (9) and (10) are the relay voltage and current respectively and as such are known. Relay current i flowing through the rail impedance causes a drop $e_2 = Zi$ and likewise the relay voltage e impressed across the rails throughout the length of the track circuit produces a leakage current $i_2 = Ge$. Zi and Ge therefore constitute the second terms of their respective series. Obviously $e_2 = Zi$ (where i is constant) increases uniformly from the relay to the transformer and its average value is therefore $\frac{e_2}{2}$ and the corresponding ballast leakage current is $i_3 = \frac{e_2}{2}G = \frac{G}{2}Zi$; likewise, it may be shown that $e_3 = \frac{Z}{2}Ge$. These last quantities thus constitute the third term of the current and voltage series respectively.

The development of the next voltage term e_4 from i_3 presents some difficulty in that we have no reason for assuming that the average value of i_3 is $\frac{i_3}{2}$; as a matter of fact, it is not, since i_3 contains the product of the two factors G and Z, varying with the length of the track circuit, and hence increases with the square of the distance from the relay. It may be demonstrated by the calculus that in any equation of the form $y = x^n$ the average value of y between the limits of y, and o is $\frac{1}{(n + 1)}$ of the maximum value of y. Therefore, the average value of i_3 above is $\frac{i_3}{3}$ and the corresponding *e.m.f.* is $e_4 = \frac{Zi_3}{3} = \frac{Z}{3}\,\frac{G}{2}\,Zi$, and likewise, $i_4 = \frac{G}{3}\,\frac{Z}{2}\,Ge$; these latter values form the fourth terms of the voltage and current series respectively, and the process may be carried out until equations (9) and (10) are entirely duplicated. It should be noted that any term in the current series is derived from the preceding term in the voltage series by multiplying by the conductance G, divided by 1, 2, 3, etc., depending on the number of the term, which is perfectly logical since it is that preceding voltage which causes the current in question to flow; conversely, any term in the voltage series is derived from the preceding term in the current series by multiplying it by Z, divided by 1, 2, 3, etc.

Comparison of center leak and distributed leak methods.

If the above terms are developed by the center leak method, in which the entire ballast conductance is considered as being concentrated at the center of the block, we find that

$$E = e + Zi + \frac{Z}{2} Ge + \frac{Z}{2} \frac{G}{2} Zi \qquad (11)$$

$$I = i + Ge + \frac{G}{2} Zi \qquad (12)$$

The first three terms of the above formulae are identical with the corresponding terms of equations (9) and (10) calculated on the distributed leak basis. The fourth term of equation (11) is however 50 per cent greater in value than the corresponding term of equation (9). The center leak method will, therefore, give sufficiently accurate results where the track circuit is short enough in length to permit all terms after the third being disregarded.

Application of track circuit formulae; examples.

Let us apply formulae (9) and (10) to two of the usual track circuit arrangements, first, considering a galvanometer relay on a steam road, and, second, a polyphase relay on an electric road. Both of these relays are of the two-element type and one of them (the galvanometer) works most economically with the currents in its track and local elements in phase or nearly so, while the other (the polyphase) works best with its track and local currents in quadrature. These examples may therefore be considered as representative; calculations for a track circuit employing a single-element relay would of course be made in exactly the same manner, the calculation and diagram as used in the case of a two-element relay being simply discontinued after the track volts, amperes and power factor at the transformer are determined for the one winding used in the case of the single-element instrument.

Galvanometer Relay: See vector diagram Fig. 20

Steam road 100-pound rails, 33 feet long, bonded with 2-40 per cent copper-clad wires.

Track circuit 5000 feet long, end fed; ballast resistance 6 ohms per 1000 feet.

Relay; track 1.7 V., 1.0 A., 0.9 P. F., on 60 cycles;
local 110 V., 0.3 A., 0.4 P. F., on 60 cycles.

Rail impedance $Z = 5 \times 0.31 = 1.55$ at 0.68 P. F. (See Table I).

Ballast conductance $G = 5 \times {}^1/_6 = 0.83$ at 1 P.F.

Relay and transformer leads to track, 100 feet No. 9 each set = 0.08 ohm.

Resistance drop in relay leads $= 1 \times 0.08 = 0.08$ volt.

$E = 1.78 + 1.55 + 1.14 + 0.33 + 0.12 + \ldots\ldots$

$I = 1.0 + 1.48 + 0.643 + 0.318 + 0.069 + \ldots\ldots$

Volts at rails opposite relay = 1.78 obtained from Fig. 20; it is the vectorial sum of the relay voltage $e = 1.7$ and the lead drop 0.08 volt, the latter being in phase with and hence parallel with the current vector $i = 1$ drawn at an angle whose P.F. = 0.9 lagging behind the relay volts $e = 1.7$.

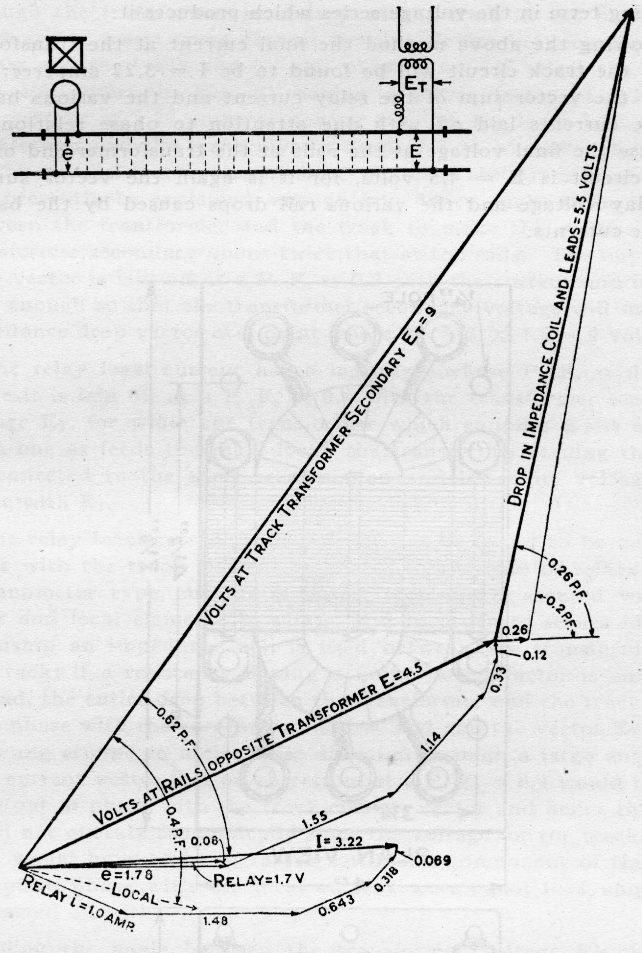

Fig. 20.
Vector Diagram for a Track Circuit Equipped with a Galvanometer Relay.

In plotting the various leakage currents and their corresponding drops in Fig. 20 it will be remembered that each term in the voltage series is obtained by multiplication of the preceding terms in the current series by *Z*; the power factor of *Z* is 0.68 and hence each term of the voltage series is laid off at a lead angle whose P. F. = cos. $\ominus$ = 0.68 using the preceding term of the current series as a base line. The ballast conductance G is, of course, non-inductive and its

P. F. = 1; hence each term in the current series is parallel with the preceding term in the voltage series which produces it.

Following the above method the final current at the transformer end of the track circuit will be found to be I = 3.22 amperes; it is simply the vector sum of the relay current and the various ballast leakage currents laid off with due attention to phase relationship. Likewise the final voltage at the rails at the transformer end of the track circuit is E = 4.5 volts, for it is again the vector sum of the relay voltage and the various rail drops caused by the ballast leakage currents.

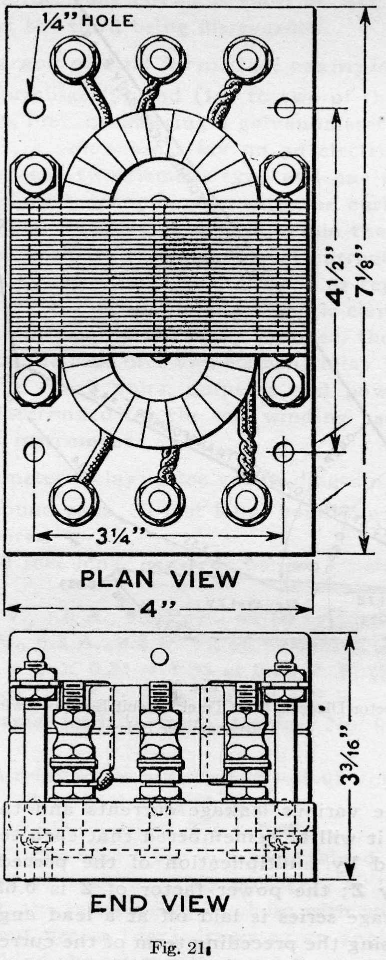

Fig. 21.
Track Impedance Coil.

To prevent the flow of an injurious short circuit current flowing through the transformer secondary with a train in the block, it is necessary to insert some current limiting device between the transformer and the track and for this purpose we will select the impedance coil shown in Fig. 21 having a power factor of 0.2. The feed current of 3.22 amperes flowing through the leads between the transformer and the track gives a drop of $3.22 \times 0.08 = 0.26$ volt, laid off parallel to the current since the leads are non-inductive. As will presently be explained, enough impedance ought to be inserted between the transformer and the track to make the voltage at the transformer secondary about twice that at the rails. The impedance drop vector is laid off at a P. F. = 0.2 with the current and is made long enough so that the transformer secondary voltage will meet the impedance drop vector at a point where $E_T = 2 \times 4.5 = 9$ volts.

The relay local current has a lag angle whose P. F. = 0.4, and hence it is laid off at a P. F. of 0.4 with the transformer secondary voltage E_T, for while the transformer which supplies E_T is not the same one as feeds the relay local, the transformer feeding the local is connected to the same transmission and hence its voltage is in phase with E_T.

The relay local current thus laid off will be found to be nearly in phase with the track element current. In the case of relays of the galvanometer type, maximum power economy is secured with the track and local elements in phase, and in order to secure ideal relationship an impedance coil is used between the transformer and the track; if a resistance having a unity power factor is employed instead, the entire drop between the transformer and the track would be in phase with the current vector $I = 3.22$ and the vector E_T would be swung around in a clockwise direction through a large angle; the local current vector laid off therefrom at a P. F. = 0.4 would then be away out of phase with the track current vector and hence the relay would not operate economically since the voltage for the track at the relay would have to be increased until that component of the track current in phase with the local current were equal to 1 ampere as we started off with.

Scaling the angle between the transformer voltage E_T and the current, we find it to be such that the cosine or P. F. = 0.62; hence, the total power with the block unoccupied is $E_T I \cos. \ominus = 9 \times 3.22 \times 0.62 = 18$ watts. With a train on the track circuit opposite the transformer, the current flowing will be equal to the transformer voltage E_T divided by the vectorial sum of the inserted impedance and the resistance of the transformer track leads. The drop in this part of the circuit as scaled from the diagrams is found to be 5.5 volts and this is due to a current $I = 3.22$ amperes; hence the combined value of the impedance and leads is $Z = \frac{E_T}{I} = \frac{5.5}{3.22} = 1.71$ ohms. With a train on the track circuit as above there will be only 1.71 ohms in series with the transformer secondary, and, neglecting the resistance of the wheels and axles of the train which is negligible, the short circuit current flowing will be $\frac{9}{1.71} = 5.26$ amperes, the corresponding power factor being 0.26 as scaled from the diagram, this being simply the power factor of the impedance and the resistance of the track leads in series. The total power with the block occupied is, therefore, $9 \times 5.26 \times 0.26 = 12.3$ watts. It is thus seen that the power with the block occupied is less than when the block is clear; this arises from the fact that the short circuit current with the block occupied is almost in quadrature with the transformer voltage due to the phase displacement produced by the impedance coil.

Polyphase Relay: See vector diagram Fig. 22

Electric road, 70-pound rails, 33 feet long, bonded to capacity.
Double-rail end fed track circuit, 8000 feet long.
Relay; track 0.15 V., 0.25 A., 0.65 P. F., on 25 cycles;
local 12 V., 0.20 A., 0.4 P. F., on 25 cycles.
Z on 25 cycles $= 8 \times 0.11 = 0.88$ at 0.52 P. F.
$G = 8 \times \frac{1}{4} = 2$ mhos for ballast leakage of 4 per 1000 feet.
Impedance bonds, 500 amperes per rail with unbalancing capacity of 150 amperes, impedance 0.31 ohm at 0.15 P. F.
Relay and transformer leads 100 feet No. 9 wire = 0.08 ohm.
Drop in relay leads $= 0.25 \times 0.08 = 0.02$ volt.
Volts opposite relay = 0.16 scaled from diagram.
Bond current at relay end $= \frac{0.16}{0.31} = 0.52$ ampere.
Total current at relay end = 0.75 ampere from diagram.
$E = 0.16 + 0.66 + 0.14 + 0.196 + 0.021 + 0.017$
$I = 0.75 + 0.32 + 0.66 + 0.093 + 0.095 + 0.008$

Referring to Fig. 22, the relay voltage 0.15 V. and the current 0.25 A. are laid off at a P. F. = 0.65 as before, and taking into account the drop in the leads of 0.02 V., laid off parallel to the relay current, the voltage at the relay end of the track circuit is found to be $e = 0.16$. At 0.16 volt the bond A takes 0.52 ampere and this current is laid off lagging at an angle corresponding to 0.15 P. F. with the track voltage e. The total current at the relay end of the track circuit is the vectorial sum of the relay current and the bond current and scales 0.75 ampere. Applying equations (9) and (10) and laying off the various ballast leakage currents and voltages listed above in exactly the same way as in the case of the galvanometer relay diagram, Fig. 20, the current I = 1.5 A. and the voltage E = 1 V. is found at the transformer end of the block. The bond B at the trans-

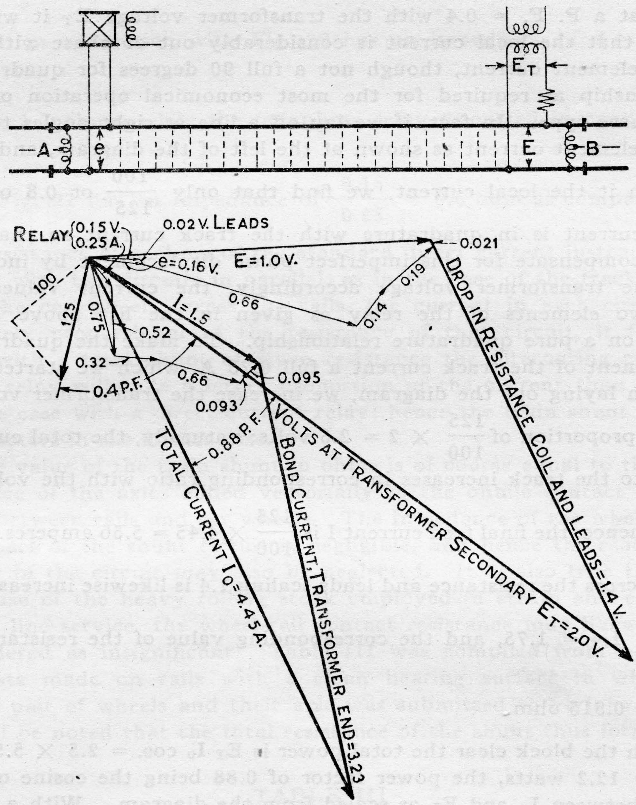

Fig. 22.
Vector Diagram for a Track Circuit Equipped with a Polyphase Relay.

former end of the block takes at 1 volt $\frac{1}{0.31}$ = 3.23 amperes and this current is laid off at a P. F. = 0.15 with the corresponding voltage E and the total current fed into the track scales 4.45 A., being the vectorial sum of the bond current 3.23 A. and the track current I = 1.5 A.; employing a resistance between the transformer and the track, the corresponding drop is laid off in phase with and parallel with the total current and with a transformer voltage of twice the track voltage, the final voltage at the transformer secondary $E_T = 2$ is obtained. The drop in the leads and resistance scales 1.4 volts and the corresponding total resistance is $\frac{1.4}{4.45}$ = 0.315 ohm.

On laying off the vector for the current in the local element of the relay at a P. F. = 0.4 with the transformer voltage E_T it will be noted that the local current is considerably out of phase with the track element current, though not a full 90 degrees for quadrature relationship as required for the most economical operation of the polyphase type. In fact, if we lay off a line at right angles to the track element current as shown at the left of the diagram, and project on it the local current, we find that only $\frac{100}{125}$ or 0.8 of the local current is in quadrature with the track current so that we must compensate for this imperfect phase displacement by increasing the transformer voltage accordingly, the current values for the two elements of the relay as given in the list above being based on a pure quadrature relationship. To make the quadrature component of the track current a full 0.25 A. which we started out with in laying out the diagram, we increase the transformer voltage in the proportion of $\frac{125}{100} \times 2 = 2.5$ volts; naturally, the total current fed into the track increases in corresponding ratio with the voltage, and, thence, the final feed current I is $\frac{125}{100} \times 4.45 = 5.56$ amperes. The drop across the resistance and leads scaling 1.4 is likewise increased to $\frac{125}{100} \times 1.4 = 1.75$, and the corresponding value of the resistance is $\frac{1.75}{5.56} = 0.315$ ohm.

With the block clear the total power is $E_T I_o$ cos. $= 2.5 \times 5.56 \times 0.88 = 12.2$ watts, the power factor of 0.88 being the cosine of the angle between I_o and E_T as scaled from the diagram. With a train on the circuit opposite the transformer, the maximum current is equal to the transformer volts divided by the total resistance between transformer and the track and is $\frac{2.5}{0.315} = 7.94$ amperes at 1 P. F. since the resistance is non-inductive. The power with the block thus occupied is $2.5 \times 7.94 \times 1 = 19.8$ watts.

It will now be apparent why a resistance was employed between the transformer and the track, for if an impedance had been used instead, the local current vector would have been nearly in phase with the track current vector and the relay would hardly have picked up even with several times its normal current, simply due to the imperfect phase displacement.

The train shunt.

In general, alternating current track relays have a much lower internal impedance than the ordinary track relays used in direct current practice; for example, the galvanometer relay which we considered in connection with Fig. 20 has an impedance of $Z = \frac{E}{I} = \frac{1.7}{1} = 1.7$ ohms, while the polyphase relay discussed in connection with Fig. 22 has an impedance of $\frac{0.15}{0.25} = 0.6$ ohm as compared to the resistance of 4 ohms of the standard direct current instruments. Since, with two circuits in parallel, as in the case of the track relay and the car wheels across the rails, the current in each circuit is inversely proportional to the resistance of that circuit, it follows that with a train shunt of given resistance the alternating current track relay will take a larger proportion of the current than would be the case with a direct current relay; hence the train shunt is not so effective in the former case.

The value of the train shunt in ohms is of course equal to the impedance of the axles added vectorially to the ohmic contact resistance between rails and car wheels. The impedance of the wheel and axle part of the shunt circuit is negligible, and hence the reactance factor in the circuit may also be neglected. It is also true that in the case of the heavy rolling stock employed in steam and electric trunk line service, the wheel-rail contact resistance may likewise be considered as insignificant. Table III was compiled from a series of tests made on rails with a clean bearing surface in which a single pair of wheels and their axle was submitted to various loads. It will be noted that the total resistance of the shunt thus formed is

TABLE III

CONTACT SURFACE OF WHEELS AND RAILS CLEAN METAL

Frequency cycles	No. of test	Total lb. weight on track	Amps. axle current	Volts across rails	Apparent ohmic resistance between rails via wheels and axle
25	1	18,700	185	0.133	0.0007
	2	23,052	175	0.13	0.0007
	3	27,404	180	0.134	0.0007
	4	36,108	180	0.14	0.0008
60	1	18,700	112.5	0.12	0.001*
	2	27,404	112.5	0.114	0.001
	3	36,108	112.5	0.11	0.00097
d.c.	1	18,700	55	0.022	0.0004
	2	27,404	56	0.021	0.00037
	3	36,108	55	0.018	0.00033

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* 2 feet 9 inches of axle gave drop of 0.048 volt. From this point (on either side) to rail average drop 0.35 volt.

practically independent of the loading; while the total shunt resistance is extremely low in all cases, it is to be noted that, as might be expected, the total apparent resistance increases with the frequency. Compared to any of these shunt resistances the impedance of the relays above given is so enormously high that the shunt may be considered as practically perfect.

It is only when rusty or dirty rail surfaces are encountered that the resistance of the train shunt becomes significant and this statement applies equally well to direct current track circuits; every signalman is familiar with the occasional difficulties experienced on heavily sanded track. Table IV indicates what the surface contact resistance may amount to, the tests having been made on a four wheel truck loaded so that the total weight on the rails was 40,900 pounds.

TABLE IV

	Clean rails		Rusty rails	
	25 cycles	60 cycles	25 cycles	60 cycles
Total amps. through axles	220	180	70	125
Volts across rails	0.232	0.1	0.82	0.37
Train shunt in ohms	0.00105	0.00055	0.0117	0.003

On steam and electric trunk lines where the rolling stock is generally heavy and the train movements are frequent enough to keep the rails clean, it may be safely assumed that the train shunt resistance is so extremely low as to be negligible. On some of the interurban trolley lines, however, where light single car trains are operated and movements are not frequent enough to keep the rails bright, the value of the train shunt must be taken into consideration; in such cases, it has been found to run much higher than the values in Table IV and since the relay ought to be shunted out to a point at least 50 per cent below its minimum shunt point, it is often customary to make electric road track circuit shunt calculations with a train shunt value of 0.064 ohm. In those cases on electric roads where it is suspected that the train shunt may be of comparatively high resistance due to light rolling stock and rusty rails resulting from infrequent train service, it is therefore generally advisable to check the track circuit calculations as described below in order to be certain that the relay will be shunted open with a train in the block.

Methods of controlling track circuit sensitiveness.

In the first place, the train shunt is least effective when the train is opposite the relay, for at that time the entire rail impedance will be in circuit between the train shunt and the track transformer with the result that the track feed current and the consequent drop in the resistance or impedance between the transformer and the track will be less than with the train opposite the transformer; then, since the voltage at the rails opposite the transformer is the vectorial difference between the transformer voltage and the drop in the resistance or reactance inserted between the transformer and the track, it follows that with the train opposite the relay, the voltage at the track opposite the transformer and in turn that opposite the relay will be greatest when the train is at the relay end of the track circuit. Since this latter is the worst condition encountered, calculations to improve the effectiveness of the track circuit should be made on this basis.

It is desirable to reduce to the lowest point possible the voltage at the relay with a train in the block, and it will be immediately apparent from the above that where the impedance or resistance (as the case may be) is inserted between the transformer and the track, there is a very effective means of controlling the voltage at the rails opposite the transformer, since, as this voltage decreases, so also will the voltage at the relay decrease. Hence, with an impedance or resistance of high value the short circuit current with a train on the block will cause a correspondingly heavy drop between the transformer and the track and as a result the track voltage opposite both transformer and relay will be low. It is customary to use sufficiently high impedance or resistance so that with the block clear the voltage at the track opposite the transformer will be about one-half that at the transformer secondary, and it was with the train shunt in mind that these values were employed in connection with Figs. 20 and 22; with such an adjustment the track voltage will generally fall to a perfectly safe figure when a train comes on the track circuit. Inserting impedance or resistance to give a transformer voltage greater than twice the track voltage will rarely be justified since after the relay is once shunted out with a large margin of safety any further increase in inserted impedance or resistance will only result in a useless waste of power.

With the aid of the vector diagrams shown in Figs. 20 and 22, it is not a difficult matter to ascertain whether the inserted impedance or resistance, determined on the basis of the transformer volts being twice the track volts with the block occupied, will insure the track relay being open with a train opposite it. It is assumed of course that all the track circuit constants are known, including the shunting point as well as the normal operating point of the relay. With a train shunt of given value across the rails at the relay, use the normal operating voltage of the relay plus the drop in the track leads as the first term e of equation (9), and considering the train shunt as an impedance bond of unity power factor, construct a diagram like Fig. 22, leaving the propulsion bonds out if a steam road track circuit is being investigated; in the case of an electric road circuit, the train shunt will simply constitute an extra bond at the relay end in multiple with the propulsion bond. The vector diagram thus obtained will of course indicate a transformer voltage considerably greater than what is actually existent as determined from the calculation with the block clear. Calling this hypothetical transformer voltage E_{TS} and the actual existent transformer voltage E_T, the volts e at the relay end of the block must be reduced in the proportion of $\frac{E_T}{E_{TS}}$; in turn the relay voltage and current will be decreased in like ratio and if with this reduction it is found that the relay current is below the shunting point of the instrument, the impedance or resistance chosen for the "block clear" condition may be considered as satisfactory. If the reduction is not sufficient, the impedance or resistance inserted between the transformer and the track will have to be arbitrarily increased until the calculations prove that the relay is effectively shunted.

Power factor triangle.

In laying out vector track circuit diagrams such as those shown in Figs. 20 and 22, the value of the various angles are given by the calculations in terms of their cosines, these being the power factors of the corresponding angles. The phase spacing of vectors is, therefore, much more easily effected through the use of a triangle marked off in cosines, such as that shown in Fig. 23, than through the employment of a protractor indicating degrees, since in the latter case the angles corresponding to the cosine would have to be looked up

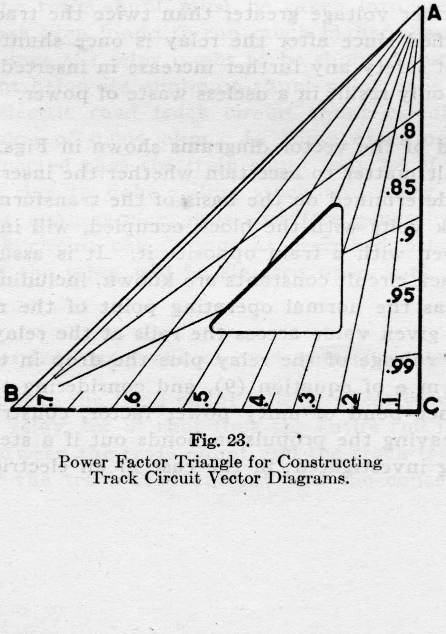

Fig. 23.
Power Factor Triangle for Constructing Track Circuit Vector Diagrams.

in a table. A transparent triangle should be used as it is often necessary to use it upside down; the lines 0.1 to 0.7, as drawn from vertex A, are laid off with a protractor at angles with base line B C corresponding to cosines of 0.1 to 0.7 as given in standard cosine tables; the lines 0.8 to 0.99, as drawn from vertex B, are laid off from base line B C, likewise at angles corresponding to cosines of 0.8 to 0.99. Considerable care should be exercised in using the triangle at first, especially when reversing it, as otherwise one may be using the complement of the angle instead of the angle itself.

Instructions for the Adjustment, Maintenance and Operation of Alternating Current Track Circuits

General.

1. The adjustment of alternating current track circuits depends on many factors, such as the type of track circuits; whether double or single rail with steam or electric traction; the length of track circuits; the ballast condition; the desired shunting values at both relay and transformer ends of the track circuit; the type of relay; and the weight of train equipment.

Voltage values.

2. Voltage at the relay track terminals should be at least 10 per cent higher than the values given on the manufacturer's tag or name plate to take care of varying ballast, bonding and power factor conditions. A check for the proper voltage to be applied across the track terminals of a relay under wet or other unavoidable conditions, is to shunt the relay, so that its contacts open half way, then remove the shunt, and if the contacts come up to the front stop, sufficient energy is passing through the track windings.

3. Voltage at the relay local terminals should be as near as possible the normal voltage given on the manufacturer's tag or name plate. If less than the normal voltage value is impressed across the local terminals, more energy will have to be used in the track element to cause proper operation, except for centrifugal frequency relays. While a reduction in the local voltage of a centrifugal frequency relay generally does not require additional energy to the track element, this reduction in the local voltage should be avoided, as it tends to increase the time of shunting.

4. When shunting two-element track relays, do not place a shunt across local terminals for this will blow the fuse protecting this circuit. Shunting of the relay should be done at the terminals of the track element.

5. Voltage at the rails opposite the transformer should be not more than one-half that at the transformer secondary under the most unfavorable track conditions. This voltage should be high enough to give the 10 per cent margin at the relay track terminals. Where reactive transformers are in use, the rail voltage should not be more than one-half the open circuit transformer secondary voltage.

Shunting sensitivity.

6. Shunting sensitivity tests of track circuits should be made by determining what is the maximum resistance that can be placed across the rails opposite the relay to cause the relay contacts to open. This can be done by finding what is the longest piece of No. 14 or No. 16 A.W.G. copper wire which, when positively connected to the bootlegs opposite the relay, or to rail clamps, will shunt the relay. Forty feet of No. 14 or 25 feet of No. 16 wire (0.1 ohm) is considered good shunting sensitivity. These shunting tests should be made under dry weather conditions when the maximum current is flowing through the relay, as the relay will shunt more easily in wet weather when less current flows through it. Shunting sensitivity tests should also be made at the rails opposite the transformer in the same manner as mentioned above. Usually the value of the resistance at this point will be higher than that obtained at the relay end of the track circuit.

Phase angle measurements.

7. To obtain the best operation of two-element track relays under adverse ballast conditions, it is suggested that phase measuring instruments be used to determine if the phase angle relations in the relay approach the ideal values. These measurements may indicate that changes should be made in adjustment to improve phase relations.

Pick-up and drop-away.

8. These values obtained in the field for two-element track relays will not generally check with the values given on the manufacturer's tag or name plate due to the fact that the manufacturer's values are for ideal phase relations.

9. When making pick-up and drop-away tests, at least three readings should be recorded for each operation and the average taken.

10. The drop-away values should not be less than a given percentage of pick-up values for the particular type of relay under test. The safe drop-away percentage values vary considerably for different types of relays and should be obtained from manufacturers or by tables shown in the Signal Section Manual giving field requirements of alternating current track relays.